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Machine Learning II
2 Exercise Sheet
Due on Wed, May 4, 14:15

Exercise 1. *Show that if two random variables X and Y are independent, then their covariance $\mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$ is zero.* **2 points**

Exercise 2. *Suppose that we have three coloured boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange, and no limes, and box g contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2, p(b) = 0.2, p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?* **2 points**

Exercise 3. *Suppose you have observed N samples $x_1, \ldots, x_N$ drawn from a Gaussian distribution. Compute the maximum likelihood estimators for the mean and variance of the data, i.e.*

$$\max_{\mu,\sigma^2} \log \prod_{n=1}^{N} p(x_n; \mu, \sigma^2)$$

where $p(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$. **3 points**

Exercise 4. *Consider a data set in which each data point $(\mathbf{x}_n, t_n)$ is associated with a weighting factor $r_n > 0$, so that the sum-of-squares error function becomes*

$$E_D(\mathbf{w}) = \frac{1}{2} \sum_n r_n (t_n - \mathbf{w}^T \Phi(\mathbf{x_n}))^2$$

Find an expression for the solution $\mathbf{w}^*$ that minimizes this error function. Give two alternative interpretations of the weighted sum-of-squares error function in terms of (i) data dependent noise variance and (ii) replicated data points. **3 points**