Exercise 1. Give a real-world example of a joint distribution $P(x, y)$ where $x$ is discrete and $y$ is continuous.

1 point

Exercise 2. What remains if I marginalize a joint distribution $P(v, w, x, y, z)$ over five variables with respect to variables $w$ and $y$? What remains if I marginalize the resulting distribution with respect to $v$?

1 point

Exercise 3. If variables $x$ and $y$ are independent and variables $x$ and $z$ are independent, does it follow that variables $y$ and $z$ are independent?

1 point

Exercise 4. Show that the following relation is true:

$$P(w, x, y, z) = P(x, y)P(z|w, x, y)P(w|x, y)$$

2 points

Exercise 5. In my pocket there are two coins. Coin 1 is unbiased, so the likelihood $P(h = 1|c = 1)$ of getting heads is 0.5 and the likelihood $P(h = 0|c = 1)$ of getting tails is also 0.5. Coin 2 is biased, so the likelihood $P(h = 1|c = 2)$ of getting heads is 0.8 and the likelihood $P(h = 0|c = 2)$ of getting tails is 0.2. I reach into my pocket and draw one of the coins at random. There is an equal prior probability I might have picked either coin. I flip the coin and observe a head. Use Bayes’ rule to compute the posterior probability that I chose coin 2.

3 points

Exercise 6. Consider a biased die where the probabilities of rolling sides $\{1, 2, 3, 4, 5, 6\}$ are $\{1/12, 1/12, 1/12, 1/12, 1/6, 1/2\}$, respectively. What is the expected value of the die? If I roll the die twice, what is the expected value of the sum of the two rolls?

2 points