

Real time survey



The screenshot shows the eduVote website interface. At the top left is the 'eduVote' logo. Below it is a large image of a lecture hall filled with students. A thought bubble above the hall contains a smartphone displaying 'A B C D' and a bar chart with three bars of increasing height. On the left side, there is a vertical navigation menu with buttons for 'eduVote - ARS', 'Downloads', 'License', 'Testimonials', and 'FAQ'. The main content area has the heading 'eduVote is an Audience Response System for the Academic Environment:'. Below this heading, there are two columns of text. The left column describes the use of ARS and lists benefits. The right column is titled 'Our Partners:'. At the bottom left is the 'S² SimpleSoft' logo. At the bottom right are links for 'T&C's', 'LEGAL NOTICE', 'CONTACT', and a German flag icon.

eduVote is an Audience Response System for the Academic Environment:

The use of ARS, also known as a TED-System or voting system, is considered very beneficial in large lecture halls and various other teaching arenas:

- Students are engaged through active participation in the material, thus increasing their attention span.
- As the students must give precise answers, they gain an awareness of where they may have knowledge gaps.
- In addition, the instructor is able to gain a quick overview of the audience's current knowledge on the subject being discussed.

In comparison with other voting systems that require proprietary hardware (e.g. a hand-held clicker), eduVote is very cost-efficient since it provides Apps that run on a student's laptop or smartphone. Thus minimizing the time and effort required to organize and distribute equipment and eradicating any purchase or maintenance costs.

In comparison to a web-based system, we take privacy extremely seriously. The eduVote server does not receive data regarding the instructor's question or the student's voting results. The question and answers are stored locally on the instructor's local machine. We are aware and respect that instructors value this control over their questions and results.

Anything else? eduVote incurs no usage-based costs! eduVote can be integrated into PowerPoint for Windows! And, since 2011, eduVote has been successfully used at a number of universities across Germany, Austria and Switzerland.

eduVote Testimonials: evaluation and feedback on eduVote can be viewed [here](#).

T&C's LEGAL NOTICE CONTACT 

<http://www.eduvote.de/en/>

Exercises:

- maximal number of points: 110
- Typically, 1 exercise sheet per week: 10 points
- Sometimes larger sheets for two subsequent weeks: 20 points

Grading policy:

- 50% of all points: eligible for the final exam (date to be determined)
 - 75% of all points: improves the exam grade by 0.3/0.4
e.g. 2.0->1.7, 1.7->1.3, 1.0->1.0
-

Today's topics

- Curve fitting (and overfitting)
 - Regularization
 - Probability and expectation
 - Normal distribution
 - Likelihood
 - Mixture of Gaussians
 - Non-parametric methods
-

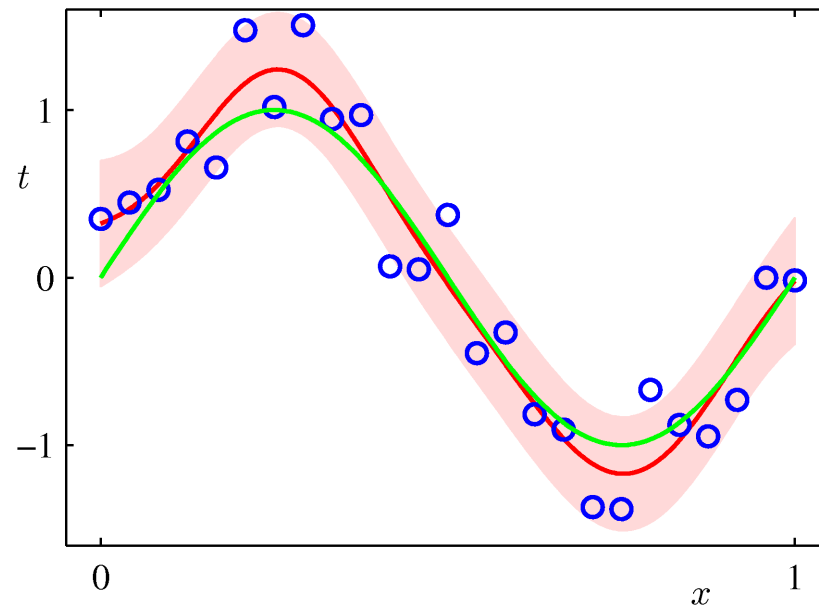
Slides modified from:
PATTERN RECOGNITION
AND MACHINE LEARNING
CHRISTOPHER M. BISHOP

and:

Computer vision: models,
learning and inference.

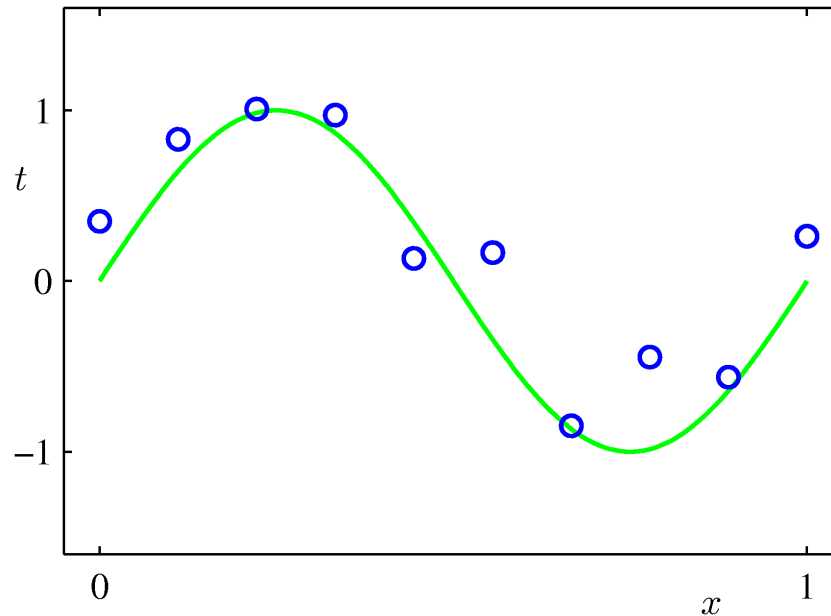
©2011 Simon J.D. Prince

Pattern recognition: an example



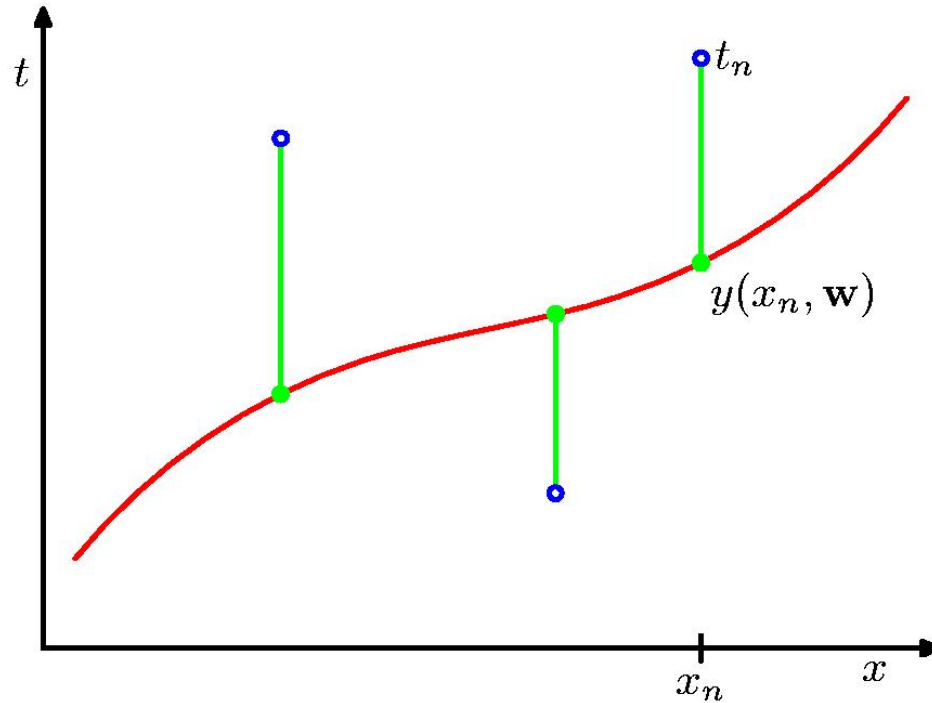
Linear Basis Function Models (1)

Example: Polynomial Curve Fitting



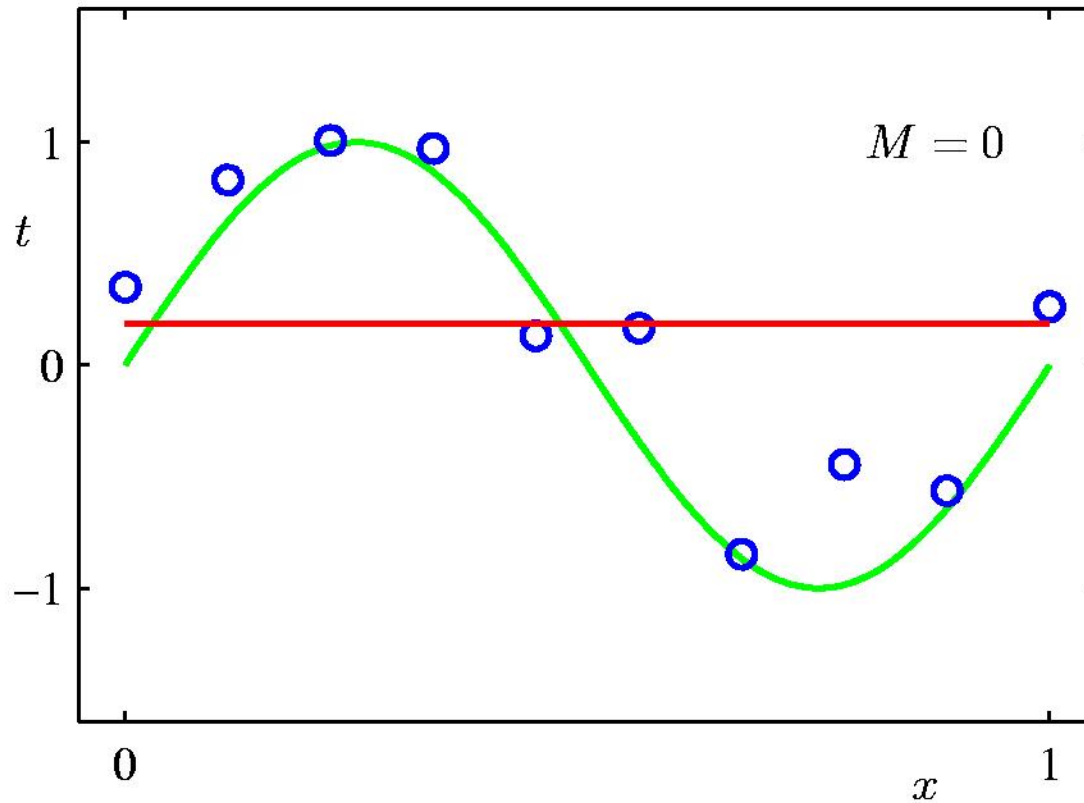
$$y(x, \mathbf{w}) = w_0 + w_1x + w_2x^2 + \dots + w_Mx^M = \sum_{j=0}^M w_jx^j$$

Sum-of-Squares Error Function

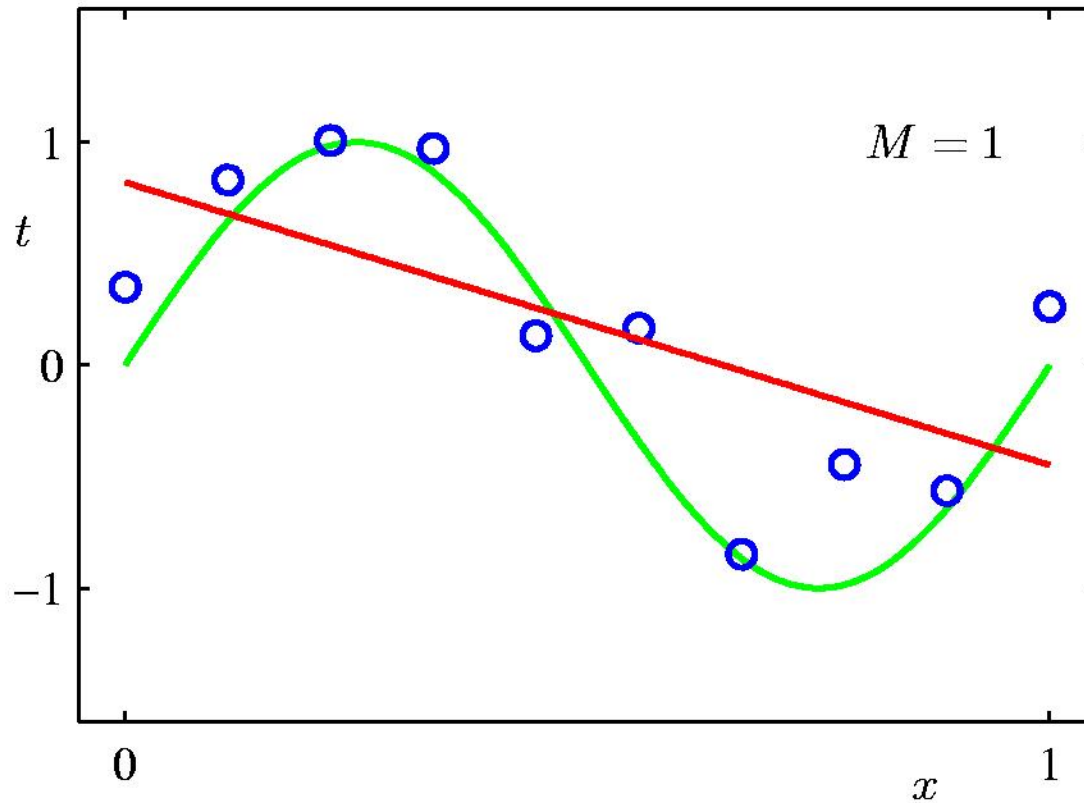


$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2$$

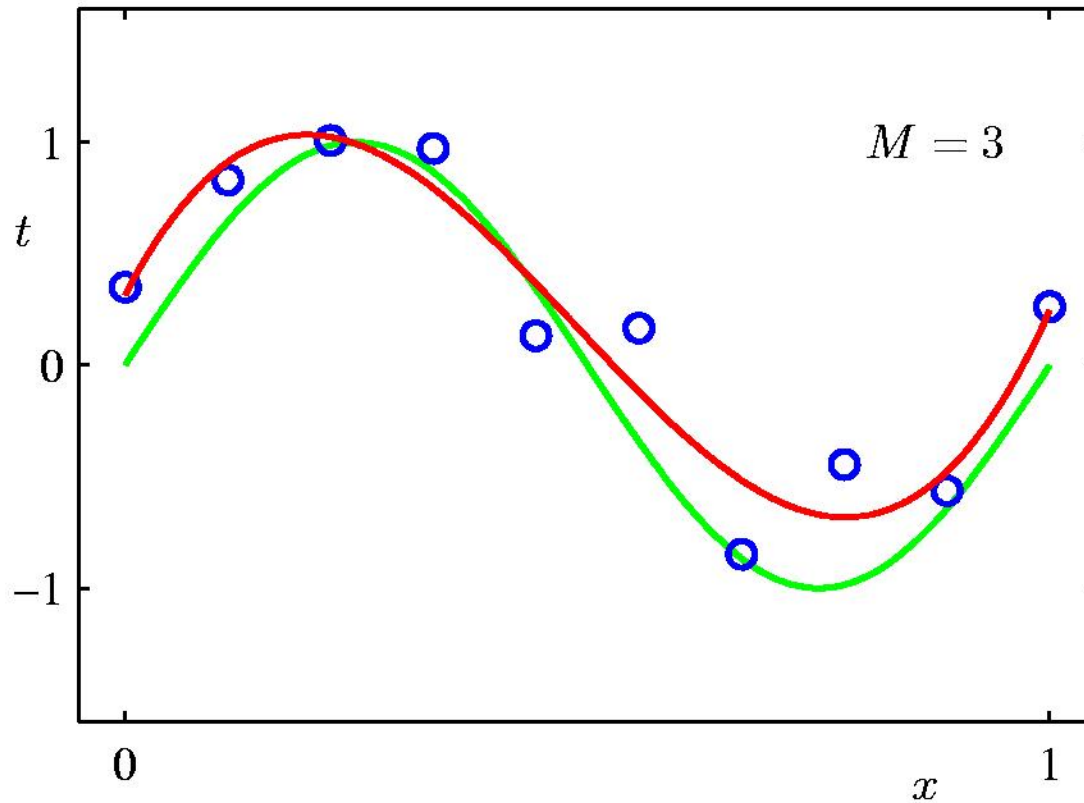
0th Order Polynomial



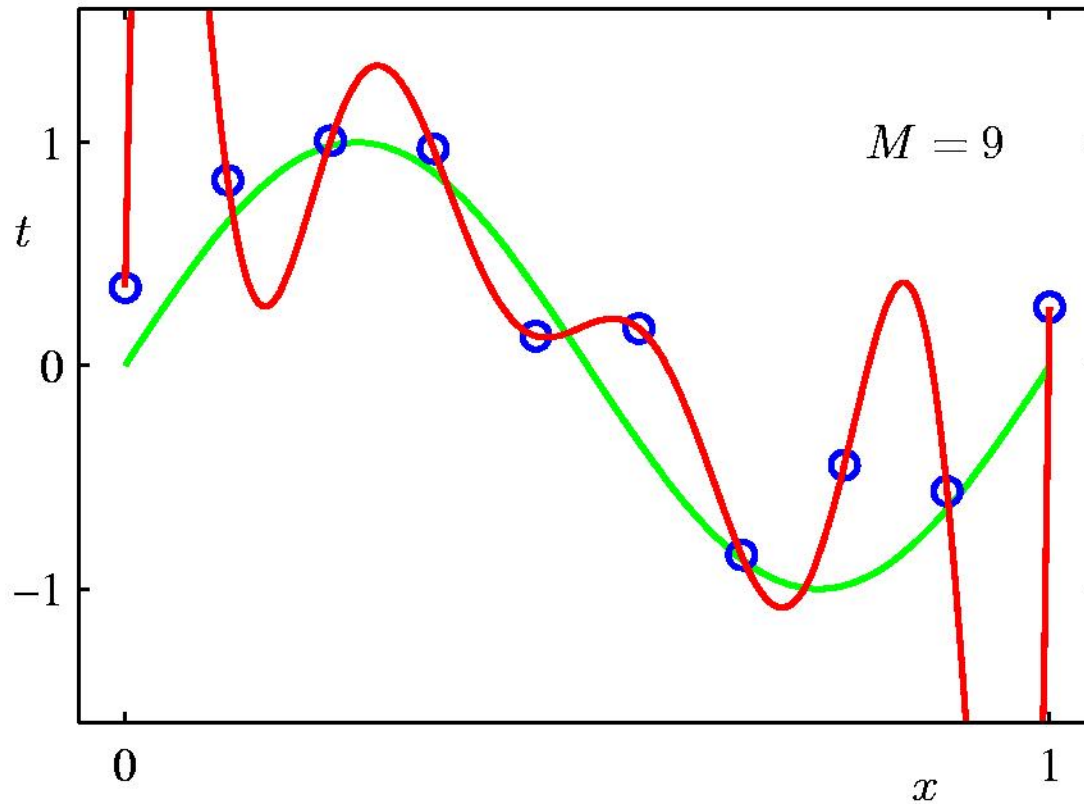
1st Order Polynomial



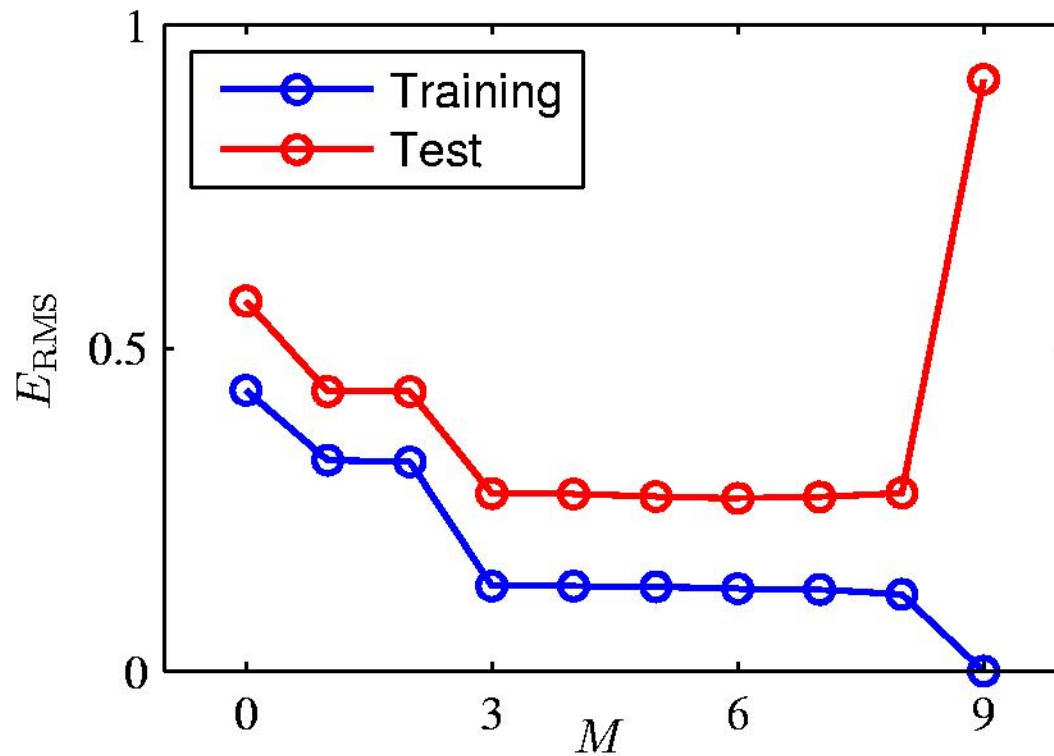
3rd Order Polynomial



9th Order Polynomial



Over-fitting



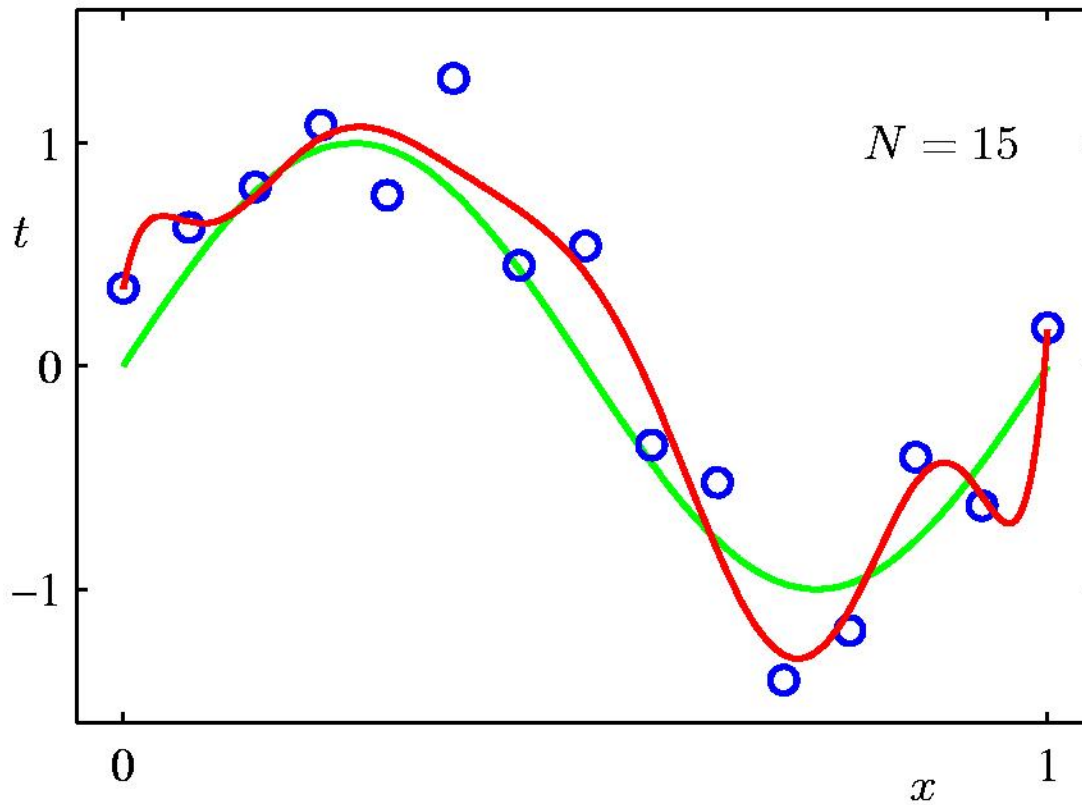
Root-Mean-Square (RMS) Error: $E_{\text{RMS}} = \sqrt{2E(\mathbf{w}^*)/N}$

Polynomial Coefficients

	$M = 0$	$M = 1$	$M = 3$	$M = 9$
w_0^*	0.19	0.82	0.31	0.35
w_1^*		-1.27	7.99	232.37
w_2^*			-25.43	-5321.83
w_3^*			17.37	48568.31
w_4^*				-231639.30
w_5^*				640042.26
w_6^*				-1061800.52
w_7^*				1042400.18
w_8^*				-557682.99
w_9^*				125201.43

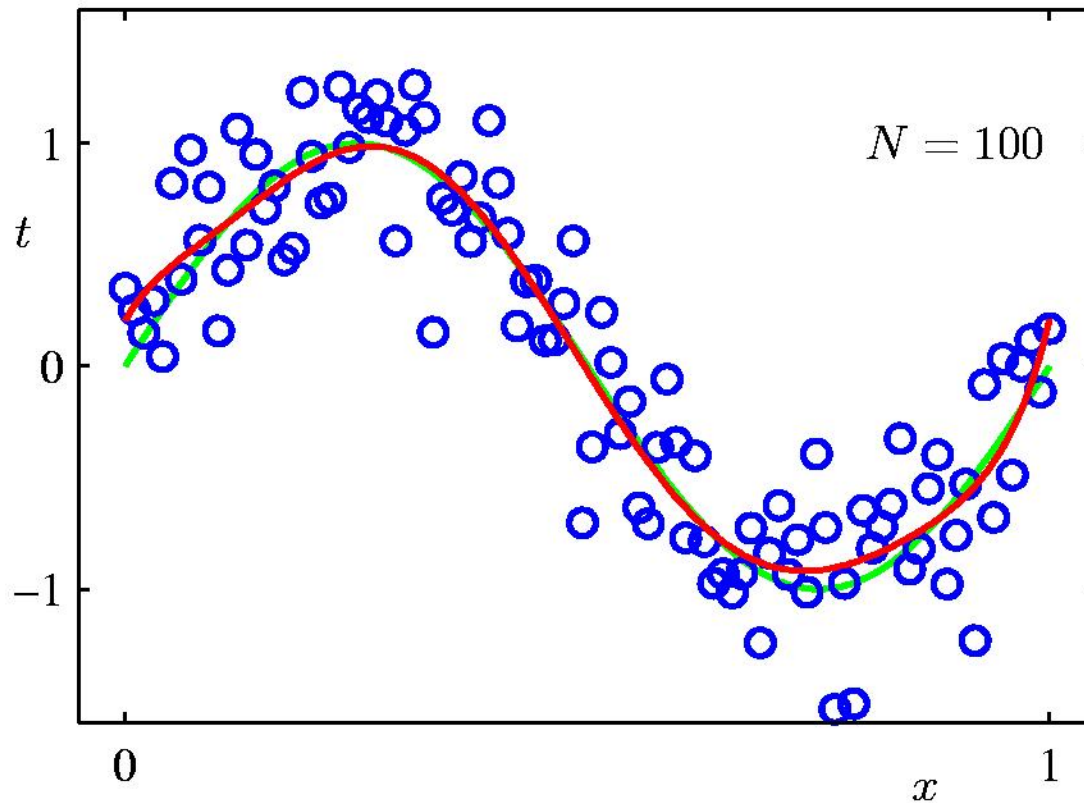
Data Set Size: $N = 15$

9th Order Polynomial



Data Set Size: $N = 100$

9th Order Polynomial

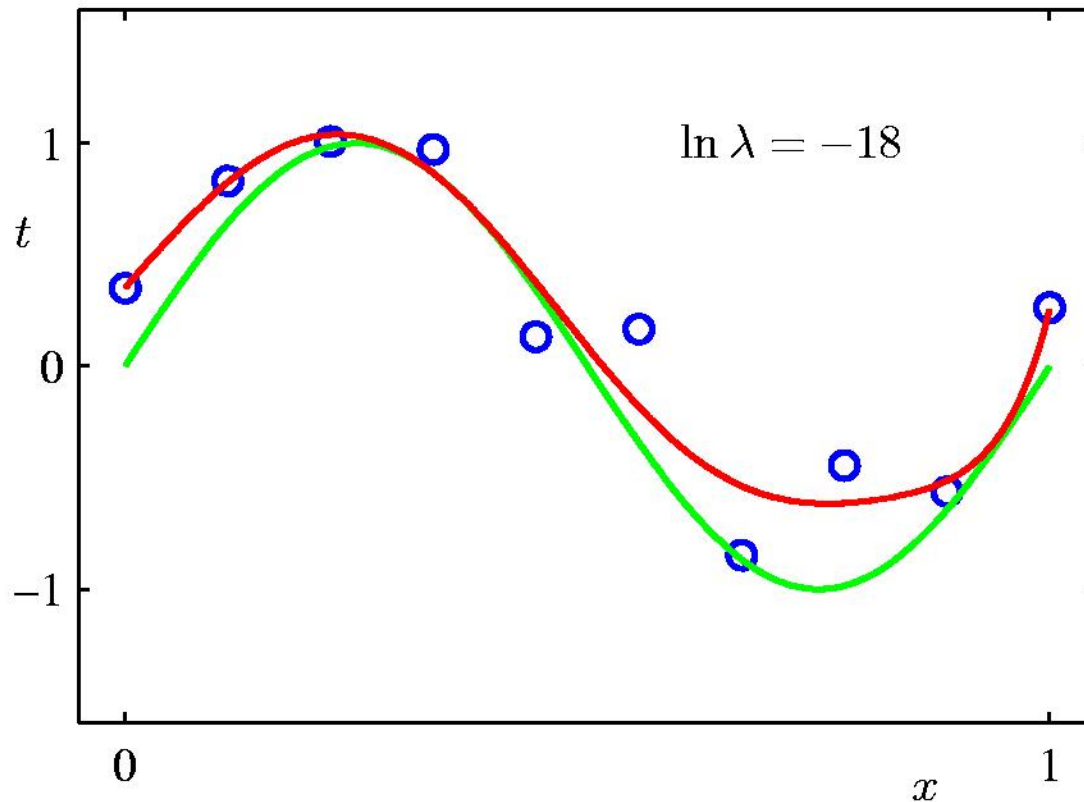


Regularization

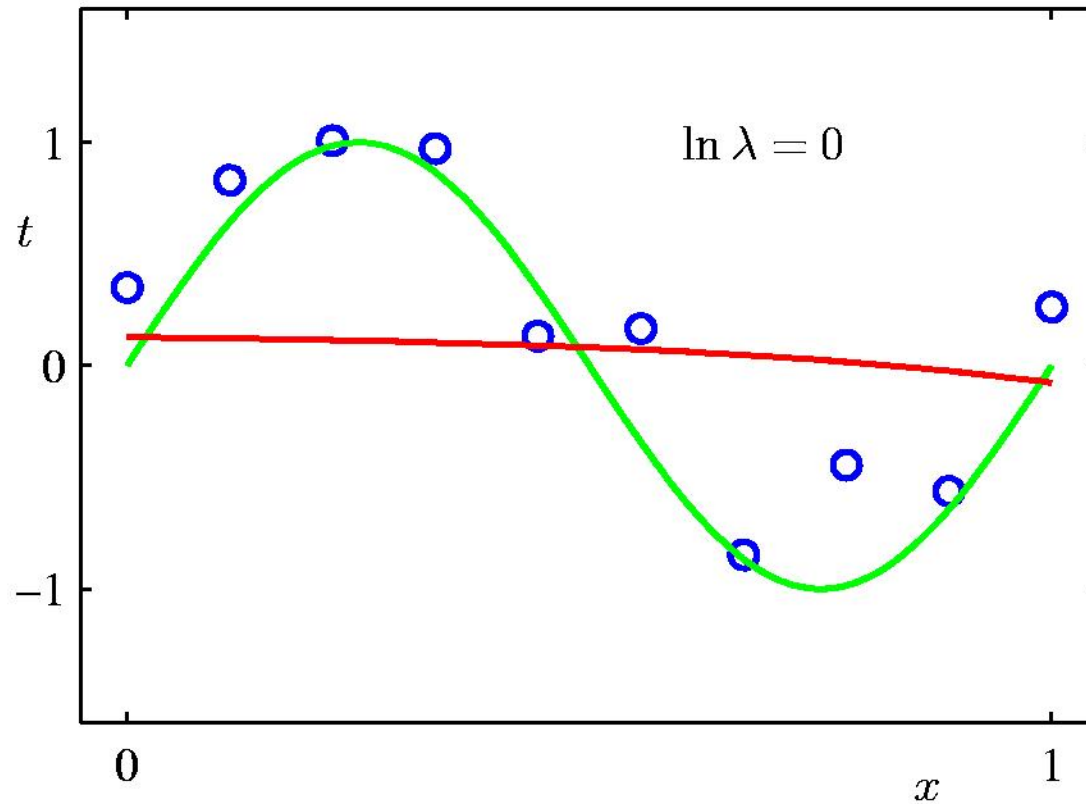
Penalize large coefficient values

$$\tilde{E}(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2 + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

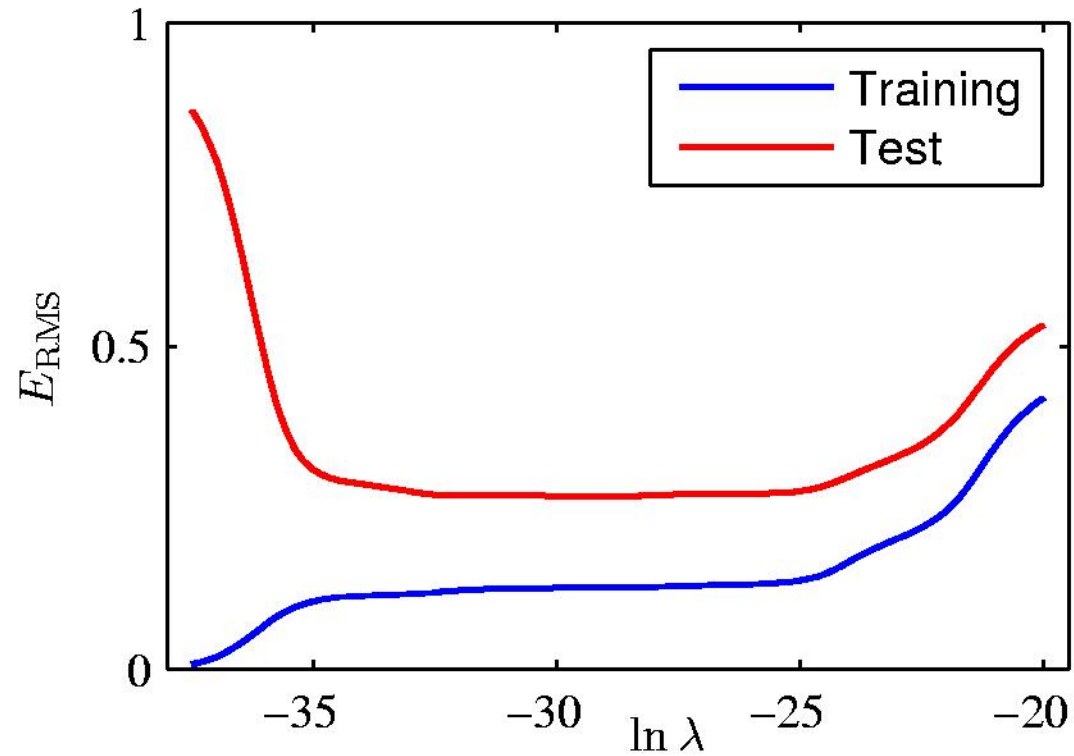
Regularization: $\ln \lambda = -18$



Regularization: $\ln \lambda = 0$

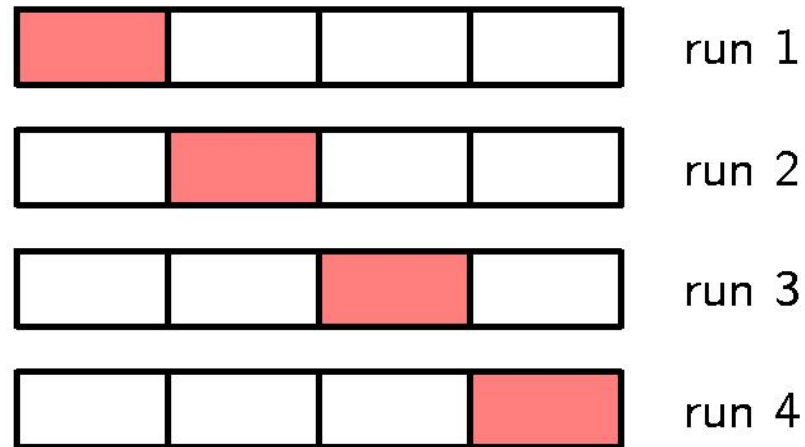


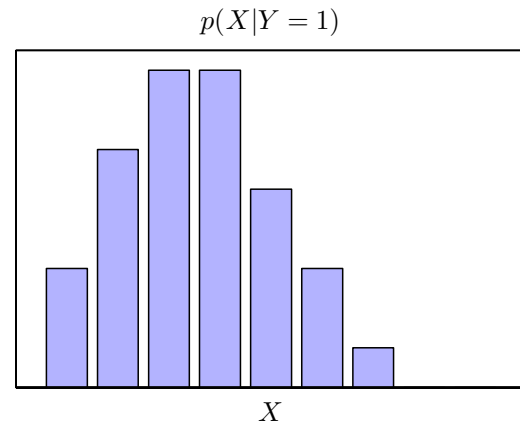
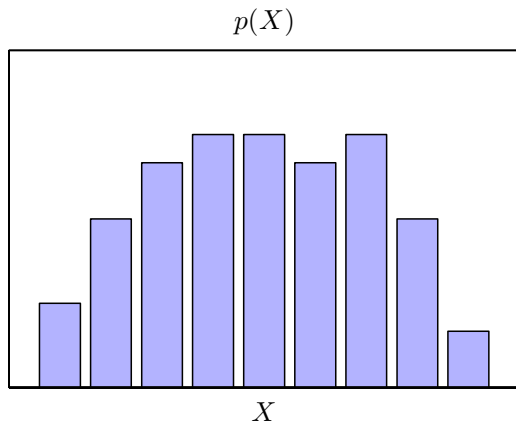
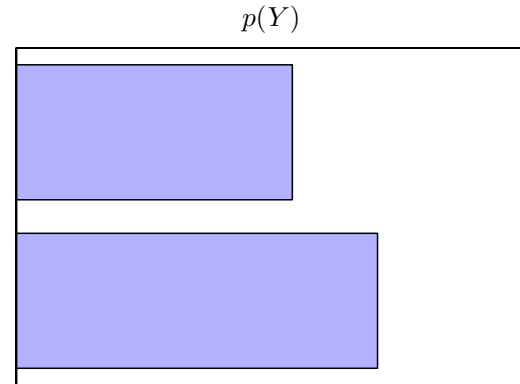
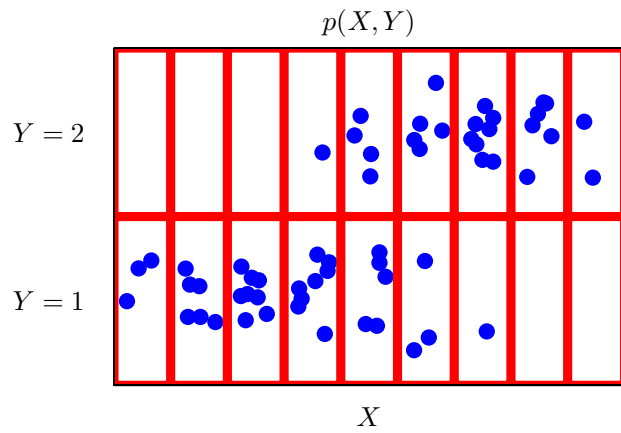
Regularization: E_{RMS} vs. $\ln \lambda$



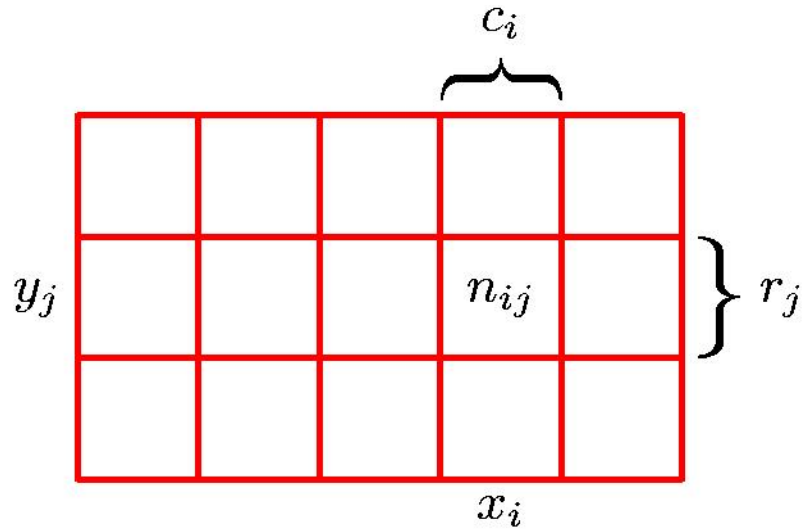
Model Selection

Cross-Validation





Probability Theory



Marginal Probability

$$p(X = x_i) = \frac{c_i}{N}.$$

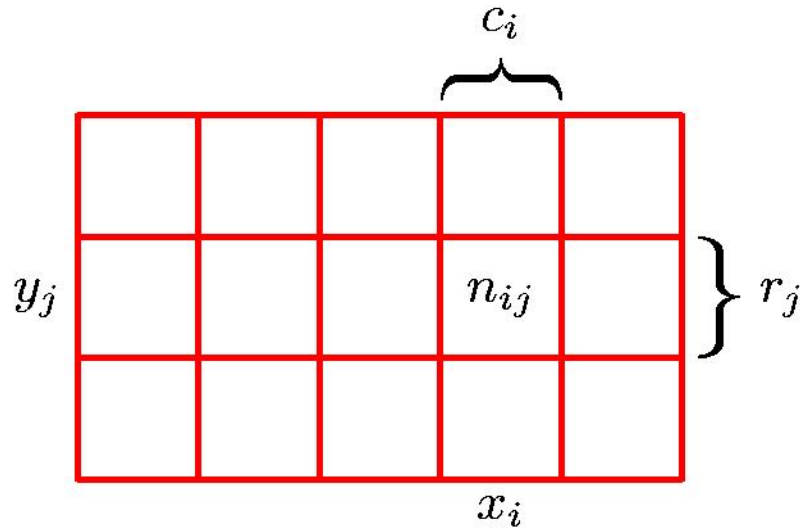
Joint Probability

$$p(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

Conditional Probability

$$p(Y = y_j | X = x_i) = \frac{n_{ij}}{c_i}$$

Probability Theory



Sum Rule

$$\begin{aligned} p(X = x_i) &= \frac{c_i}{N} = \frac{1}{N} \sum_{j=1}^L n_{ij} \\ &= \sum_{j=1}^L p(X = x_i, Y = y_j) \end{aligned}$$

Product Rule

$$\begin{aligned} p(X = x_i, Y = y_j) &= \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N} \\ &= p(Y = y_j | X = x_i) p(X = x_i) \end{aligned}$$

The Rules of Probability

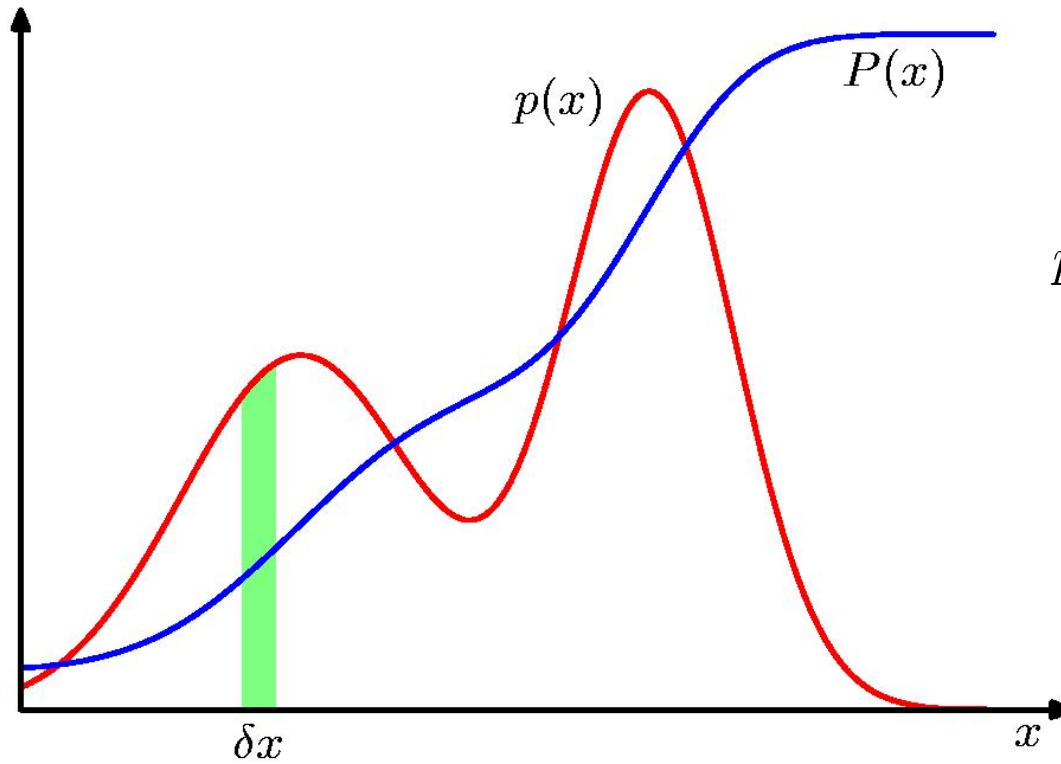
Sum Rule

$$p(X) = \sum_Y p(X, Y)$$

Product Rule

$$p(X, Y) = p(Y|X)p(X)$$

Probability Densities



$$p(x \in (a, b)) = \int_a^b p(x) dx$$

$$P(z) = \int_{-\infty}^z p(x) dx$$

$$p(x) \geq 0$$

$$\int_{-\infty}^{\infty} p(x) dx = 1$$

Bayes' Theorem

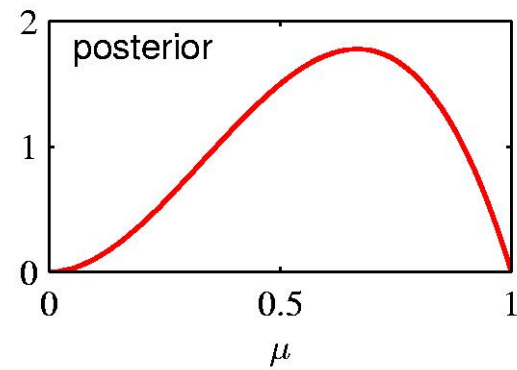
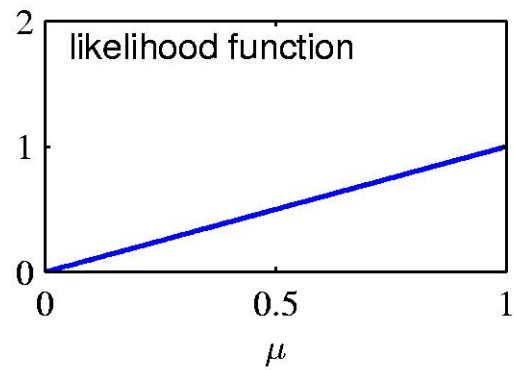
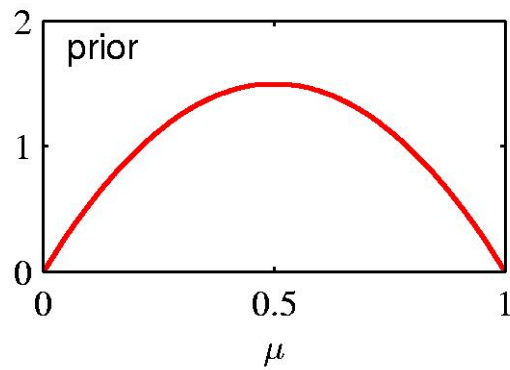
$$p(Y|X) = \frac{p(X|Y)p(Y)}{p(X)}$$

$$p(X) = \sum_Y p(X|Y)p(Y)$$

posterior \propto likelihood \times prior

Use $p(X, Y) = p(Y|X)p(X)$

Prior · Likelihood = Posterior



Expectations


$$\mathbb{E}[f] = \sum_x p(x) f(x)$$

$$\mathbb{E}[f] = \int p(x) f(x) dx$$

Expectations

$$\mathbb{E}[f] = \sum_x p(x) f(x)$$

$$\mathbb{E}[f] = \int p(x) f(x) dx$$

$$\mathbb{E}_x[f|y] = \sum_x p(x|y) f(x)$$


Conditional Expectation
(discrete)

$$\mathbb{E}[f] \simeq \frac{1}{N} \sum_{n=1}^N f(x_n)$$

Approximate Expectation
(discrete and continuous)

Variances and Covariances

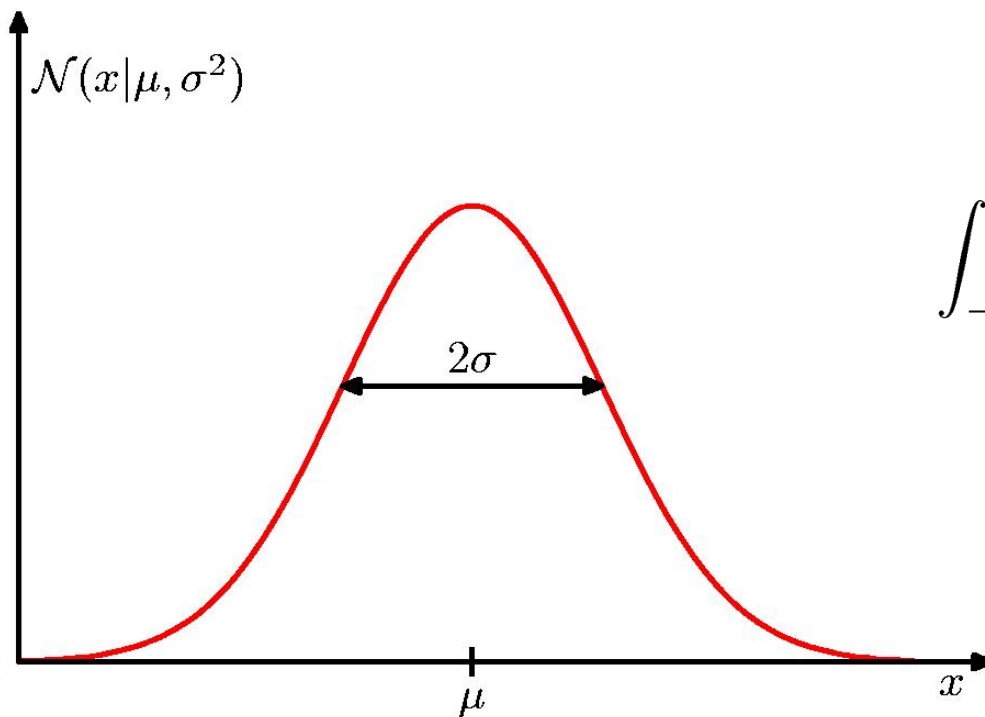
$$\text{var}[f] = \mathbb{E} \left[(f(x) - \mathbb{E}[f(x)])^2 \right] = \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2$$

$$\begin{aligned} \text{cov}[x, y] &= \mathbb{E}_{x,y} [\{x - \mathbb{E}[x]\} \{y - \mathbb{E}[y]\}] \\ &= \mathbb{E}_{x,y}[xy] - \mathbb{E}[x]\mathbb{E}[y] \end{aligned}$$

$$\begin{aligned} \text{cov}[\mathbf{x}, \mathbf{y}] &= \mathbb{E}_{\mathbf{x},\mathbf{y}} [\{\mathbf{x} - \mathbb{E}[\mathbf{x}]\} \{\mathbf{y}^T - \mathbb{E}[\mathbf{y}^T]\}] \\ &= \mathbb{E}_{\mathbf{x},\mathbf{y}}[\mathbf{x}\mathbf{y}^T] - \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{y}^T] \end{aligned}$$

The Gaussian Distribution

$$\mathcal{N}(x|\mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x - \mu)^2\right\}$$



$$\mathcal{N}(x|\mu, \sigma^2) > 0$$

$$\int_{-\infty}^{\infty} \mathcal{N}(x|\mu, \sigma^2) dx = 1$$

Gaussian Mean and Variance

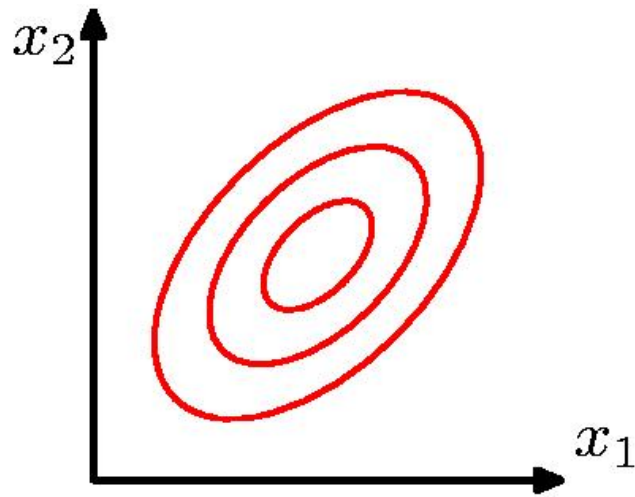
$$\mathbb{E}[x] = \int_{-\infty}^{\infty} \mathcal{N}(x|\mu, \sigma^2) x \, dx = \mu$$

$$\mathbb{E}[x^2] = \int_{-\infty}^{\infty} \mathcal{N}(x|\mu, \sigma^2) x^2 \, dx = \mu^2 + \sigma^2$$

$$\text{var}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2 = \sigma^2$$

The Multivariate Gaussian

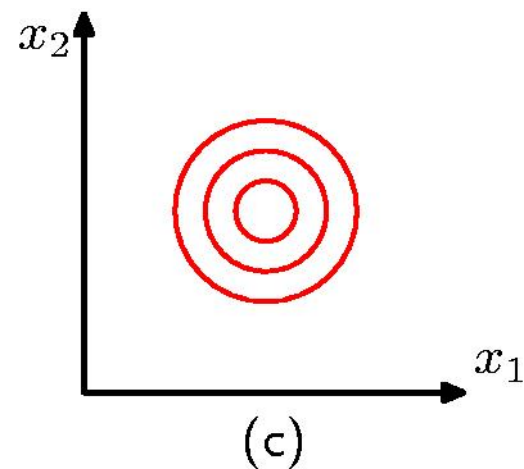
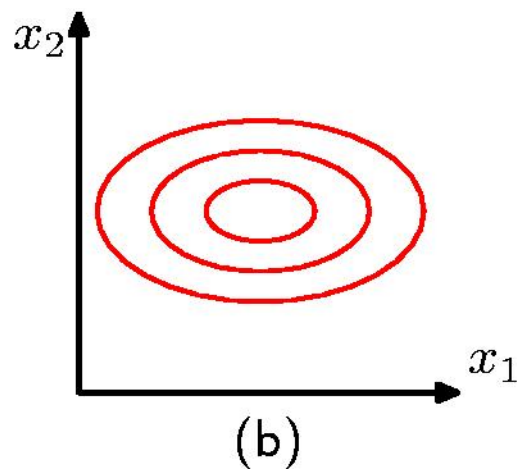
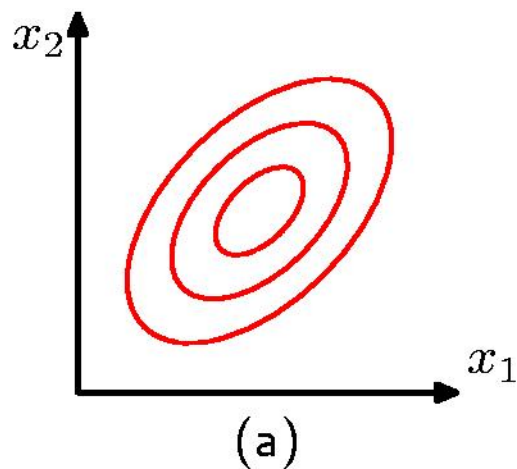
$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\boldsymbol{\Sigma}|^{1/2}} \exp \left\{ -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \right\}$$



Moments of the Multivariate Gaussian (2)

$$\mathbb{E}[\mathbf{x}\mathbf{x}^T] = \boldsymbol{\mu}\boldsymbol{\mu}^T + \boldsymbol{\Sigma}$$

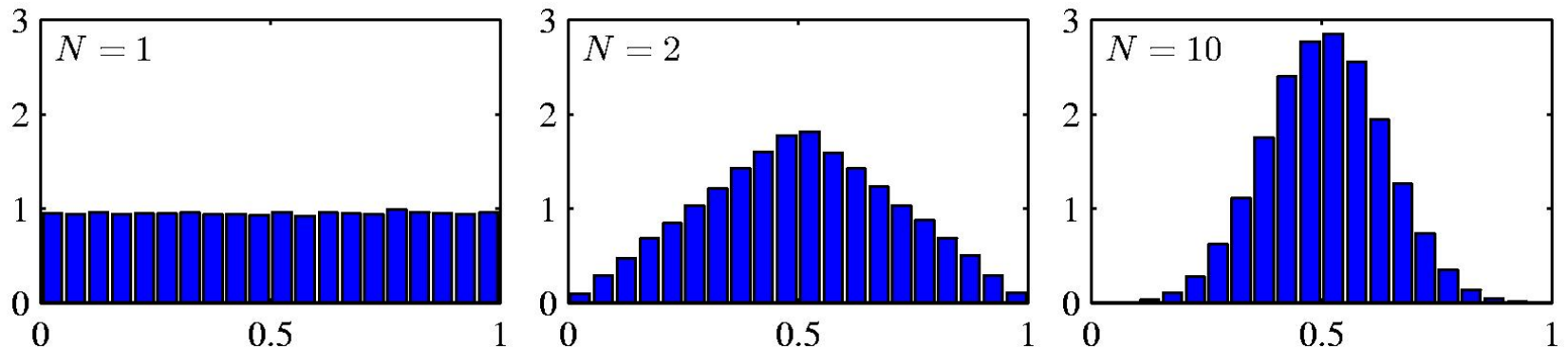
$$\text{cov}[\mathbf{x}] = \mathbb{E} [(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T] = \boldsymbol{\Sigma}$$



Central Limit Theorem

The distribution of the sum of N i.i.d. random variables becomes increasingly Gaussian as N grows.

Example: N uniform $[0, 1]$ random variables.



Partitioned Conditionals and Marginals

